PRE-AP CALCULUS Name:________________________

Symmetry Tests and Odd/Even Functions

Tests for symmetry are done on relations, not necessarily functions. There are three tests, ***x*-axis**, ***y*-axis** and **origin**. Mr. O'Brien will explain the three tests in class. What you are doing is writing a well-structured argument to show if a relation is symmetric with respect to the *x*-axis, *y*-axis, origin, or has no symmetry.

Your presentation (argument) should have the following elements (in this order):

1. State what test you are showing.
2. Show the correct substitution – just substitute, no simplification.
3. Simplify so you can compare – one side must be exactly the same as the original.
4. Compare, using words.
5. Conclude, again, using words.

Here is one of the examples we will do in class:

Problem: Determine if the relation $y - x = x^3$ has any symmetries.

Solution:

Test for x-axis symmetry:

$-y - x = x^3$ since the right sides are the same but the left sides are not, this relation is not symmetric with respect to the x-axis.

Test for y-axis symmetry:

$y - (-x) = (-x)^3 \rightarrow y + x = -x^3 \rightarrow -y - x = x^3$ since the right sides are the same but the left sides are not, this relation is not symmetric with respect to the y-axis.

Test for origin symmetry:

$-y - (-x) = (-x)^3 \rightarrow -y + x = -x^3 \rightarrow y - x = x^3$ since this is the same as the original, this relation is symmetric with respect to the origin.

If you examine the structured and well-presented solution above you will have a good model for your homework. Below, I show you what is happening in each of the 5 steps for the test for *y*-axis symmetry:

1. State the test: "Test for *y*-axis symmetry:"
2. Substitute – in this case, substitute $-x$ for x: " $y - (-x) = (-x)^3$ "
3. Simplify: " $y + x = -x^3 \rightarrow -y - x = x^3$ "
4. Compare: "since the right sides are the same but the left sides are not,"
5. Conclude: "this relation is not symmetric with respect to the *y*-axis"

Test each of the following for any symmetries that may exist and **write a structured and well-presented solution for two:**

1. $3x^2 - 2y^2 = 5$	2. $y^3 = 2x^2$	3. $xy - x^2 = 3$
4. $4x - y^2 = 2$	5. $xy = 6$	6. $y = 5 - x^2$

The next concept is a bit like the first, but is used on functions. The problem is to find out if a function can be classified as and "even" function, an "odd" function or neither. **Even functions have *y*-axis symmetry** and **odd functions have origin symmetry**.

The definition of **even** or **odd** functions is as follows:

If *f* is even, then $f(-x)=f(x)$. If *f* is odd, then $f(-x)=-f(x)$. [Can you see why?]

So, you see the key to the analysis is to form $f(-x)$ and compare it with $f(x)$.

As with the symmetry arguments, you need to present a well-constructed, complete solution.

Example 1: Determine if $f(x)=-3x$ is an even function, an odd function or neither.
Solution: $f(-x)=-3(-x)=3x\neq f(x)$ since $f(-x)\neq f(x)$ then *f* is not even.
$f(-x)=-3(-x)=3x=-(-3x)=-f(x)$ since $f(-x)=-f(x)$ then *f* is odd.

Notice that the second line of the solution is exactly the same as the first up to the 3rd item after $f(-x)$, when the expression changes to $-(-3x)$. To test for an odd function, you factor out a negative at this step and then compare.

Another way to tell the symmetry in Example 1 is to think graphically- you know that this is a line passing through the origin and hence has origin symmetry.

Example 2: Determine if $g(t)=5t-6$ is an even function, an odd function or neither.
Solution: $g(-t)=5(-t)-6=-5t-6\neq g(t)$ since $g(-t)\neq g(t)$ then *g* is not even.
$=-(5t+6)\neq -g(t)$ since $g(-t)\neq -g(t)$ then *g* is not odd.
Final conclusion: *g* is neither even nor odd.

Again, thinking graphically, we knew that this line with a slope of 5 and *y*-intercept of 6 was neither odd nor even.

Example 3: Determine if $h(x)=7-3x^2$ is an even function, an odd function or neither.
Solution: $h(-x)=7-3(-x)^2=7-3x^2=h(x)$ since $h(-x)=h(x)$ then *h* is even.

[Note: of course, a function can be only even or odd or neither, so once you've found what it is, you can stop– as you see in Example 3.]

Finally, from a graphical perspective, this vertically translated and dilated parabola is clearly even.

The Khan Academy has a nice 12-minute video explaining even and odd functions. Determine if each function below is even, odd or neither and **write a structured and well-presented solution for two:**

7. $f(x)=2x-4x^3$
8. $g(x)=x^2-3x+5$
9. $h(x)=4x^2-7$
10. $p(x)=1+\sqrt{|x|}$

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